Here's a structured response addressing each part of the question, aiming for the clarity and rigor expected in a graduate-level statistics exam:

\*\*(a) Interpretation of the Slope:\*\*

The slope of the regression line represents the estimated change in the percentage of nitrogen removed for a one-foot increase in the width of the grass buffer strip. A positive slope indicates that wider buffer strips are associated with a greater percentage of nitrogen removal. The magnitude of the slope quantifies the strength of this association. For example, a slope of 2 would suggest that for every additional foot of width, the percentage of nitrogen removed increases by approximately 2%.

\*\*(b) Prediction and Extrapolation:\*\*

While the model might be suitable for predicting nitrogen removal within the range of observed buffer strip widths (approximately 6 to 13 feet), it would be unwise to extrapolate to widths between 0 and 30 feet. Extrapolation beyond the observed data range is risky because the relationship between buffer strip width and nitrogen removal might not remain linear outside this range. There could be threshold effects, diminishing returns at wider widths, or other non-linear behaviors that are not captured by the linear model. Using the model outside of its observed range increases the risk of making significant prediction errors.

\*\*(c) Sampling Distribution of the Sample Mean (6-foot strips):\*\*

Assuming the amount of nitrogen removed for 6-foot buffer strips is normally distributed with a mean μ and standard deviation σ, the sampling distribution of the sample mean (x̄) of four observations will also be approximately normal. Its mean will be equal to the population mean μ, and its standard deviation (standard error) will be σ/√4 = σ/2. The central limit theorem justifies this approximation, even if the underlying population distribution is not perfectly normal, due to the sample size (n=4) being greater than 2. However, the accuracy of the normal approximation depends on the degree to which the underlying population is itself normal.

\*\*(d) 95% Confidence Interval for the Sample Mean (6-foot strips):\*\*

To construct a 95% confidence interval for the sample mean of the observations from four 6-foot buffer strips, we would use the following formula:

x̄ ± t<sub>(0.025, 3)</sub> \* (s/2)

Where:

\* x̄ is the sample mean of the four observations.

\* t<sub>(0.025, 3)</sub> is the critical t-value for a two-tailed test with α = 0.05 and 3 degrees of freedom (n-1). This can be obtained from a t-table.

\* s is the sample standard deviation of the four observations.

This interval provides a range of values that has a 95% probability of containing the true sample mean of nitrogen removal for 6-foot buffer strips in future samples, given the observed sample in question.

\*\*(e) Comparing Study Plans:\*\*

The second study plan (right-hand plot) would likely provide a better estimator of the slope of the regression line. This is because the observations in the second plan show less scatter around the regression line than those in the first. The data points in the second study plan appear closer to the line, suggesting a stronger relationship between the predictor and response variables. A smaller residual variance implies greater precision in estimating the slope. Therefore, the slope estimated from the second plan will likely have a smaller standard error and be more efficient.

\*\*(f) Checking the Linearity Assumption:\*\*

To check the linearity assumption, instead of selecting only two buffer strip widths, researchers could employ a more sophisticated sampling design. A useful approach would be to select buffer strips with a wider range of widths, incorporating a design of experiment approach, for example, including at least three different widths with several replicates at each width to allow for a test of lack of fit in a regression model. This would enable them to visually inspect a scatter plot of nitrogen removal percentage against buffer strip width, observe possible patterns of curvature, and perform formal statistical tests (e.g., lack-of-fit test in regression) to ascertain the adequacy of the linear model. A non-linear model may be needed if the data does not support a linear relationship.